On the determination of the Epoch-Correction of an adopted system of Right Ascensions of Clock Stars from observations of the Sun, including the effects of personalities; and applications of the results to the determination of the errors of the Tables of the Sun and Moon. By E. J. Stone, M.A., F.R.S., Radcliffe Observer.

There are few points in practical astronomical work which involve more delicate considerations than those connected with the determination of the absolute corrections required by the right ascensions of our Clock Stars, and the determination of the errors of the Tables of the Sun in longitude. I propose to give in the present paper some results on this subject to which I have been led, and some practical applications.

In an investigation of this kind it is necessary to accurately define the physical meanings of the symbols employed; and the notation which I have adopted is as follows:—

θ, λ, ω denote the longitude and latitude of the Sun's centre, and the "Obliquity of the Ecliptic," computed from a set of Solar Tables with an assigned value of the variable, t.

$\theta+\delta\theta$, $\lambda+\delta\lambda$, $\omega+\delta\omega$ denote the corresponding exact values.

α, Δ denote the corresponding tabular R.A. and N.P.D. computed in the usual way from the values of θ, λ, ω.

α', Δ' are the R.A. and N.P.D. of the Sun's centre as found directly from observation; and to which the variable, t, with which the tabular places are computed, is assumed to correspond.

$\alpha'+x$, $\Delta'+y$ are the exact values of the R.A. and N.P.D.

θ', λ' are the longitude and latitude of the Sun's centre from observation as found from α', Δ', and ω.

$\theta'+\delta\theta'$, $\lambda'+\delta\lambda'$ are the exact values of the longitude and latitude of the Sun's centre.

We have, therefore, $\theta+\delta\theta=\theta'+\delta\theta'$; $\lambda+\delta\lambda=\lambda'+\delta\lambda'$ identically.

The systematic errors in R.A., x, with which we shall be principally concerned arise from

(1) Errors in the adopted R.A. of the clock stars.

(2) Differences in personalities in the observations of transits of stars and limbs of the Sun.

The systematic errors in N.P.D., y, will be chiefly due to

(3) Errors in the adopted latitudes of the observing stations.

(4) Uncorrected division-errors of the circles.

(5) Personalities on the part of the observers.

(6) Errors of the adopted refraction tables.

We shall neglect the squares and products of the small angles $\delta\theta$, δ, λ, $\delta\lambda$, $\delta\omega$, x, and y; and we then have

$$(7)\ \alpha' + x = \alpha + \frac{\cos\omega}{\sin^2\Delta}\,\delta\theta - \frac{\sin\omega}{\sin^2\Delta}\sin\theta\,.\,\cos\theta\,.\,\delta\omega - \frac{\sin\omega}{\sin^2\Delta}\,.\,\cos\theta\,.\,\delta\lambda,$$

$$(8)\ \Delta' + y = \Delta - \frac{\sin\omega}{\sin\Delta}\,.\,\cos\theta\,.\,\delta\theta - \frac{\cos\omega}{\sin\Delta}\,.\,\sin\theta\,.\,\delta\omega - \frac{\cos\omega}{\sin\Delta}\,.\,\delta\lambda,$$

or, putting $\alpha - \alpha' = f$, $\Delta - \Delta' = e$, we have

$$(9)\ x = f + \frac{\cos\omega}{\sin^2\Delta}\,.\,\delta\theta - \frac{\sin\omega}{\sin^2\Delta}\,.\,\sin\theta\,.\,\cos\theta\,.\,\delta\omega - \frac{\sin\omega}{\sin^2\Delta}\,.\,\cos\theta\,.\,\delta\lambda,$$

$$(10)\ y = e - \frac{\sin\omega}{\sin\Delta}\cos\theta\,.\,\delta\theta - \frac{\cos\omega}{\sin\Delta}\,.\,\sin\theta\,.\,\delta\omega - \frac{\cos\omega}{\sin\Delta}\,.\,\delta\lambda,$$

where $\frac{f''}{15}$ and e'' will be the errors of the Solar Tables in R.A. and N.P.D., as found directly from observation.

It is these tabular errors in R.A. and N.P.D., and the errors in longitude and latitude deduced from them, which are given in the published results of the Observatories at which solar observations are made.

The values of x, y, $\delta\omega$, $\delta\lambda$, and $\delta\theta$ have to be found from a discussion of equations of the form (9) and (10), or from others which are deducible from them; and one of the chief practical difficulties of the problem of finding these and the other constants involved in the expressions of the solar coordinates is connected with the necessity for the separation of any constant correction, x, required by the observed R.A. from any existing constant error, or slowly changing errors, in the Solar Tables in longitude. The equations

$$(11)\ \delta\lambda + x\sin\omega\,.\,\cos\theta + \delta\omega\sin\theta + y\frac{\cos\omega}{\sin\Delta} = f\,.\,\sin\omega\,.\,\cos\theta + e\frac{\cos\omega}{\sin\Delta},$$

$$(12)\ \delta\theta = -f\,.\,\cos\omega + e\frac{\sin\omega}{\sin\Delta}\,.\,\cos\theta + x\cos\omega - y\frac{\sin\omega}{\sin\Delta}\,.\,\cos\theta,$$

which are directly deduced from (9) and (10), will be found more convenient for our purpose than those equations.

If the Errors of the Tables in Ecliptic Polar Distance and Longitude, as deduced from the observed R.A. and N.P.D., and the tabular obliquity are denoted by E and L respectively, we have

$$\mathrm{E} = f\,.\,\sin\omega\,.\,\cos\theta + e\frac{\cos\omega}{\sin\Delta};\quad \mathrm{L} = f\cos\omega - e\frac{\sin\omega}{\sin\Delta}\,.\,\sin\theta;$$

and, therefore,

$$(13)\qquad \delta\lambda + x\,.\,\sin\omega\,.\,\cos\theta + \delta\omega\,.\,\sin\theta + y\frac{\cos\omega}{\sin\Delta} = \mathrm{E},$$

$$(14)\qquad \delta\theta = -\mathrm{L} + x\,.\,\cos\omega - y\frac{\sin\omega}{\sin\Delta}\,.\,\cos\theta.$$

The equations of the form (13) are independent of $\delta\theta$; and because such is the case we are able to separate the constant, x, from any existing constant error of the Solar Tables in longitude by finding x from (13). We need not discuss here the cause or causes which may give rise to such errors.

The necessity of treating the equations (9) and (10) simultaneously in order to determine x and $\delta\theta$ has been frequently overlooked; and erroneous results have sometimes been obtained by the violation of this condition. If we denote the mean error in longitude of the Solar Tables in use for the observations of the year by $\delta\theta_0$; the use of the equation (9) independently of (10) would give sensibly the value $x' = x - \dfrac{\delta\theta_0}{\cos\omega}$ instead of x; and the use of x' instead of x in the equation (14) would give the errors of the Tables in longitude sensibly cleared from any existing mean error, $\delta\theta_0$, whatever Solar Tables were in use: for

$$\delta\theta = -\mathrm{L} + x\cos\omega - y\frac{\sin\omega}{\sin\Delta}\cdot\cos\theta = -\mathrm{L} + x'\cdot\cos\omega + \delta\theta_0 - y\frac{\sin\omega}{\sin\Delta}\cdot\cos\theta$$

$$\therefore\ \delta\theta - \delta\theta_0 = -\mathrm{L} + x'\ \cos\omega - y\frac{\sin\omega}{\sin\Delta}\cdot\cos\theta.$$

Before, however, we can separate the angles $\lambda + \delta\lambda$ and $\omega + \delta\omega$, we have to define what is meant by the plane of the Ecliptic.

The Ecliptic in practical astronomical work is selected so that the Sun's centre merely oscillates above and below that plane; and the expression for the Sun's latitude $\lambda + \delta\lambda$, when measured from the "Ecliptic," is an exceedingly small angle which contains no terms of the form

$$(15) \qquad \mathrm{C} + \mathrm{A}\sin.\theta + \mathrm{B}.\cos\theta$$

where C, A, and B are *constants*.

The small changes of latitude expressed by terms of this form, and the small secular changes which exist when the plane of reference is a fixed plane, are in practice thus thrown upon the longitude of the node and inclination of the moving plane of reference, the "Ecliptic."

On account of the importance of the equations (13) and (14), it may perhaps be worth while to obtain them directly without the introduction of the errors of the Tables in R.A. and N.P.D. We have with the adopted notation

$$(16)\ \sin(\lambda' + \delta\lambda') = \cos(\omega + \delta\omega)\cos(\Delta' + y) - \sin(\omega + \delta\omega)\sin(\Delta' + y).\sin(\alpha' + x)$$

$$(17)\ \tan(\theta' + \delta\theta') = \sin(\omega + \delta\omega)\cot(\Delta' + y).\sec(\alpha' + x) + \cos(\omega + \delta\omega).\tan(\alpha' + x)$$

$$(18) \qquad \sin\lambda' = \cos\omega.\cos\Delta' - \sin\omega.\sin\Delta'\sin\alpha'$$

$$(19) \qquad \tan\theta' = \sin\omega.\cot\Delta'.\sec\alpha' + \cos\omega.\tan\alpha'.$$

In these equations the only tabular quantity is ω.

These equations lead at once to the following

(20) $$\delta\lambda' = -\sin\omega \cdot \cos\theta' \cdot x - \delta\omega \cdot \sin\theta' - y\frac{\cos\omega}{\sin\Delta'},$$

(21) $$\delta\theta' = x\cos\omega - y\frac{\sin\omega}{\sin\Delta'} \cdot \cos\theta'.$$

But we have $\lambda + \delta\lambda = \lambda' + \delta\lambda'$: $\theta + \delta\vartheta = \theta' + \delta\vartheta'$ identically.

We can find λ' and θ' from (18) and (19), and then by a direct comparison between them and the tabular quantities λ and θ, we have

$$\theta - \theta' = \text{L, and} \left(\frac{\pi}{2} - \lambda\right) - \left(\frac{\pi}{2} - \lambda'\right) = \lambda' - \lambda = \text{E}$$

$$\therefore \delta\lambda' = \delta\lambda - \text{E} \qquad \delta\theta' = \delta\theta + \text{L};$$

and, as before,

(13) $$\delta\lambda + x \cdot \sin\omega \cdot \cos\theta' + \delta\omega \cdot \sin\theta' + y\frac{\cos\omega}{\sin\Delta'} = \text{E}$$

(14) $$\delta\theta = -\text{L} + x\cos\omega - y\frac{\sin\omega}{\sin\Delta'} \cdot \cos\theta'.$$

In the equation (14) the term $x\cos\omega$ introduces a constant error into our determinations of the corrections required by the Tables in longitude, and the term $-y\frac{\sin\omega}{\sin\Delta}\cos\theta'$ affects the determination of the longitude of the perigee. It would appear that these corrections, if determinable with any close approximation to their true values, should be applied before attempts are made to correct the errors of the Tables.

When the equation (13) is thus found, it perhaps brings more clearly before the mind that, since the expression for the latitude $\lambda + \delta\lambda$ contains no terms of the form (15), the errors in Ecliptic Polar Distance, E, can contain the effects of no terms of this form except those due to the errors in R.A., N.P.D. and obliquity, x, y and $\delta\omega$; and that any required *constant* corrections x, $\delta\omega$, and y can thus be found from the equation

(22) $$x\sin\omega\cos\theta + \delta\omega \cdot \sin\theta + \frac{y\cos\omega}{\sin\Delta} = \text{E}$$

by confining our attention to the effects of terms of the required form and suitably grouping the errors in Ecliptic Polar Distance; but the existence of any sensible errors, $\delta\lambda$, will to some extent increase the errors of the determinations of the constants x, $\delta\omega$, and y as thus found from observation.

If all the observations during a year were made by the same person, we might assume without serious error that x and y would be constants so far as the personalities of the observer were concerned: and, therefore, if we included the mean effects of the errors of the refraction tables and uncorrected division-errors with those of the personalities of the observer in N.P.D.,

we should have x and y constants for all the observations of a year.

A discussion of the equations (22) when the observations of the Sun were fairly distributed over the year would then lead to fairly approximate values of the constants x, $\delta\omega$, and y. But, on account of x being multiplied by a small factor 0·398; the existence of small errors, $\delta\lambda$, which have been neglected; and the magnitude of the chance errors of observation in the case of the Sun; the probable errors of the determinations of the important constant, x, will not, even in the case of a single observer, be inconsiderable.

But in practice it is difficult to secure the condition that all the observations of the year should be made by the same person, and if more than one person be engaged on the observations we have to meet the difficulty that the constants, x, are not the same for all observers.

The plan which has been followed at Greenwich for many years, and which I have adopted at Oxford, has been to ignore the differences between the constants x and y due to personal peculiarities; and thus to obtain an approximation to the average values of the constants for the observers, and of the mean error of the Solar Tables in longitude. If the observations were thoroughly well distributed amongst the different observers over the whole year, and numerous enough to destroy the effects of chance errors of observation, the results thus found would be close approximations to the truth; but in practice the required distribution is rarely secured, and the probable errors of the determination of the required constants for each year are thus greatly increased. The effects upon the errors of the tables in longitude of the discordances between the values of x thus found for different years without change of instrument, right ascensions of clock stars, or observers, are comparable in magnitude with the errors of the Solar Tables now in use; and, because such is the case, it has not been usual to directly apply the corrections x, y, and $\delta\omega$, found for each year, to the errors in ecliptic polar distance and longitude deduced from the uncorrected observations in R.A. and N.P.D. But the means corrections for $\frac{x''}{15}$ found from the observations of several years are applied as a correction to the R.A. of the fundamental stars every few years when an independent standard catalogue is formed; corrections for $\delta\omega$ are applied at intervals when the correction has become sufficiently large to rénder this desirable, and the effects of the constant y are usually neglected.

This method of dealing with the difficulty certainly prevents any serious accumulation of errors either in the right ascensions of the stars or in the errors of the solar tables; but it can hardly be considered as perfectly satisfactory.

The personal differences in observing transits of stars and of the Sun between the three observers at Oxford happen to be

well marked. This affords facilities for testing the practicability of separating the personalities of the observers from the errors of the solar tables; and I have, therefore, separated the observations of the Sun made by Messrs. Wickham, Robinson, and Bellamy, 1884–1891. The values of the constants thus found for each observer from the observations of the different years must be expected to be rather discordant from the paucity of the observations and the unfavourable distribution of the observations in some of the years; but the mean results for several years should be fairly reliable. At all events, I have thought the point worth a trial, and the results will be found in Table (a).

Radcliffe Observations of the Sun, 1884–1891.

TABLE (a).

Year.	No. of Obs.	(x . sin ω).	$\delta\omega$.	y.	δa.	Error of Longitude (Tabular *minus* Observed).	Error of Longitude (Tab.—Obsd.) corrected for mean values of x for each observer.
		Observer W.					
		"	"	"	s	"	"
1884	23	+0·717	+0·785	−0·112	+0·120	+1·436	+0·422
1885	32	+0·349	+0·140	−0·226	+0·058	+1·641	+0·627
1886	24	+0·538	+0·342	−0·309	+0·090	+1·694	+0·680
1887	18	+0·552	+0·650	−0·507	+0·092	+2·064	+1·050
1888	13	−0·264	+0·718	−0·305	−0·044	−0·061	−1·075
1889	10	−0·176	+1·170	−0·061	−0·030	+1·475	+0·461
1890	20	+0·104	+0·693	−0·009	+0·017	+1·026	+0·012
1891	22	+1·094	+0·145	−0·383	+0·183	+2·150	+1·136
Weighted Means	162	+0·441	+0·497	−0·244	+0·074	+1·513	+0·499
		Observer R.					
1884	31	−0·032	−0·081	−0·296	−0·005	−0·033	+1·027
1885	27	−0·702	+0·532	−0·520	−0·117	−0·758	+0·302
1886	25	−0·282	+0·775	−0·135	−0·047	−0·702	+0·358
1887	21	−0·526	+0·198	−0·336	−0·088	−0·011	+1·049
1888	10	−0·466	−0·816	−0·182	−0·078	−0·048	+1·012
1889	26	−0·780	−0·155	−0·048	−0·131	−1·522	−0·462
1890	24	−0·814	+0·775	−0·325	−0·136	−1·403	−0·343
1891	30	−0·228	+0·158	−0·467	−0·038	−0·320	+0·740
Weighted Means	194	−0·461	+0·240	−0·302	−0·077	−0·632	+0·428

Year.	No. of Obs.	(x . sin ω).	$\delta\omega$.	y.	δa.	Error of Longitude (Tabular *minus* Observed).	Error of Longitude (Tab.–Obsd.) corrected for mean values of x for each observer.
			Observer F.B.				
		$''$	$''$	$''$	s	$''$	$''$
1884	33	+0·528	+0·146	−0·250	+0·088	+1·137	+0·270
1885	29	+0·815	+0·072	+0·100	+0·136	+1·801	+0·934
1886	20	+0·412	+0·004	−0·053	+0·069	+1·987	+1·120
1887	19	−0·407	−0·535	−0·267	−0·068	+1·627	+0·760
1888	12	−0·335	+0·203	−0·231	−0·056	+1·924	+1·057
1889	24	−0·312	+0·725	−0·222	−0·052	+1·384	+0·517
1890	11	+1·351	−0·157	+0·322	+0·226	+1·131	+0·264
1891	25	+0·813	+0·769	−0·560	+0·136	+2·127	+1·260
Weighted Means	173	+0·377	+0·197	−0·174	+0·063	+1·632	+0·765

The systematic character of the differences between the errors of the tables in longitude deduced directly from the observations of Messrs. Wickham, Robinson, and Bellamy is sufficiently pronounced to be recognised in the results of a single year. The mean errors of the tables in longitude for the years 1884–1891 are

(23) W. = $+1''\cdot513$; R. = $-0''\cdot632$; F.B. = $+1''\cdot632$;

and the weighted mean is $+0''\cdot765$.

The corresponding values of the constant, y, are

(24) W. = $-0''\cdot244$; R. = $-0''\cdot302$; F.B. = $-0''\cdot174$.

These values of y agree fairly well with the weighted mean $-0''\cdot250$, and they show that no very strongly marked personalities exist in the observations of the Sun in N.P.D.

The corresponding values of $\delta\omega$ are

(25) W. = $+0''\cdot497$; R. = $+0''\cdot240$; F.B. = $+0''\cdot197$;

and these corrections to the tabular obliquity are also fairly accordant with the weighted mean $+0''\cdot280$.

But the values of (x sin ω) indicate the existence of well-marked differences in the habits of the observers in taking transits of stars and one, or both, of the limbs of the Sun.

The mean values of (x sin ω) for the different observers are

(26) W. = $+0''\cdot441$; R. = $-0''\cdot461$; F.B. = $+0''\cdot377$;

and for x

W. = $+1''\cdot108$; R. = $-1''\cdot157$; F.B. = $+0''\cdot946$;

which show that the R.A. of the Sun as directly found by each observer requires to be corrected for

$$W. = +0^s\cdot074\,;\ R. = -0^s\cdot077\,;\ F.B. = +0^s\cdot063\,;$$

whilst the personal equations for transit observations of stars referred to the mean habit during the period under discussion are

$$W. = +0^s\cdot15\,;\ R. = -0^s\cdot37\,;\ F.B. = +0^s\cdot22.$$

It may be mentioned here that all the transit observations at the Radcliffe Observatory are made by the "eye-and-ear" method. The personal equations for stars are large, but the personal equation of Mr. McClellan, who succeeded Mr. Bellamy in 1892, falls about half-way between W. and R.

The differences in the errors of the solar tables in longitude for the different observers given in (23) are due to differences in personalities in observing transits of stars and of limbs of the Sun. When we apply the corrections, x, found for the three observers to their corresponding errors in longitude, see equation (14), we have for the mean errors of the tables in longitude, or $(-\delta\theta)$, at the epoch 1888·0,

$$(27) \qquad W. = +0''\cdot499\,;\ R. = +0''\cdot428\,;\ F.B. = +0''\cdot765$$

and the weighted mean is $+0''\cdot560$.

The extreme difference of the results (27) only amounts to $0''\cdot34$, whilst for the results (23) the extreme difference is $2''\cdot26$.

The results (27) are certainly within the chance errors of their determination, which include those of the constant $x \sin \omega$ multiplied by 2·3; whilst the results (23) are discordant beyond any possible chance errors of their determination or the effects of the errors of the tables in longitude. The personal corrections in time required by the right ascensions of the Sun for the different observers

$$W. = +0^s\cdot074\,;\ R. = -0^s\cdot077\,;\ F.B. = +0^s\cdot063$$

have been found quite independently of any existing errors of the tables in longitude; and, although liable to sensible chance errors of determination, they must be regarded as approximate determinations of the absolute corrections required by the right ascensions of the Sun found by the three Oxford observers with the R.A. of clock stars in use.

If we apply these corrections to the observed R.A. of the different observers, the "errors of the tables," both in ecliptic polar distance and longitude, will be less dependent on the distribution of the observations during each year among the observers; and we may be able to obtain more accurate values of the constants x', y, and $\delta\omega$ by a rediscussion of the errors of ecliptic polar distance. In this rediscussion the symbols y and $\delta\omega$ will denote the same physical quantities as before, but x' will not

denote the same quantity as x. The equations connecting the x and x' for the different observers are :
For

$$\text{W.}, x = +1''\cdot108 + x'\ ;\ \text{R.} = -1''\cdot157 + x'\ ;\ \text{F.B.} = +0''\cdot946 + x'.$$

In this rediscussion the constants x' will be assumed the same for each observer.

The results thus obtained for x', y, and $\delta\omega$ are given in the following table (β), and those found from the system hitherto adopted of neglecting the personalities are given in table (γ).

Table (β).

Year.	No. of Obs.	(x'. sin ω.)	$\delta\omega$.	y.	$\delta\alpha$.	Error of Long. (Tabular *minus* Observed).	Error of Long. (Tab.−Obsd.) corrected for mean values of x'.
		"	"	"	s	"	"
1884	87	+0·201	+0·204	−0·187	+0·034	+0·580	+0·594
1885	88	+0·092	+0·198	−0·186	+0·015	+0·632	+0·646
1886	69	+0·128	+0·452	−0·216	+0·022	+0·690	+0·704
1887	58	−0·174	+0·180	−0·440	−0·029	+0·953	+0·967
1888	35	−0·659	+0·122	−0·225	−0·110	+0·253	+0·267
1889	60	−0·551	+0·447	−0·166	−0·092	+0·085	+0·099
1890	55	−0·083	+0·560	−0·045	−0·014	−0·094	−0·080
1891	77	+0·432	+0·318	−0·451	+0·072	+1·021	+1·035
Weighted Means	529 (= sum)	−0·006	+0·308	−0·242	−0·001	+0·560	+0·574

Table (γ).

Year.	No. of Obs.	(x. sin ω.)	$\delta\omega$.	y.	$\delta\alpha$.	Error of Long. (Tabular *minus* Observed).	Error of Long. (Tab.−Obsd.) corrected for mean values of x.
		"	"	"	s	"	"
1884	87	+0·294	+0·260	−0·256	+0·049	+0·799	+0·624
1885	88	+0·233	+0·166	−0·157	+0·039	+0·961	+0·786
1886	69	+0·220	+0·450	−0·190	+0·037	+0·911	+0·736
1887	58	−0·100	+0·077	−0·422	−0·017	+1·169	+0·994
1888	35	−0·511	+0·034	−0·120	−0·086	+0·623	+0·448
1889	60	−0·557	+0·467	−0·224	−0·093	+0·140	−0·035
1890	55	−0·055	+0·463	−0·087	−0·009	−0·013	−0·188
1891	77	+0·509	+0·266	−0·468	+0·085	+1·180	+1·005
Weighted Means	529 (= sum)	+0·076	+0·280	−0·250	+0·013	+0·765	+0·591

The results of column 7 and not column 8 are usually given as the errors of the tables in longitude in the published results of the Observatories.

The results, Table (β), obtained by this rediscussion are in close agreement, as they should be, with those found by a separation of the observations of the Sun made by the different observers; and they differ but little from those obtained by neglecting the differences between the values of x for the observers and trusting to the chance of a sufficiently uniform distribution of the observations amongst the observers over the year to destroy the effects of personality on the average values of the constants for all the observers.

The errors of Le Verrier's solar tables in longitude, or $-\delta\theta$ after the corrections for the mean values of x or x' have been applied (see equation 14), are as follows:—

Epoch 1888·0.

Table (α).—Weighted mean of separate individual results $= +0''\cdot560$.

Table (β).—Rediscussion with personal corrections $= +0''\cdot560 + 0''\cdot006 \times 2\cdot3 = +0''\cdot574$.

Table (γ).—Method hitherto adopted, but with correction for x applied $= +0''\cdot765 - 0''\cdot076 \times 2\cdot3 = +0''\cdot590$.

The close agreement of the results (γ), after correction for x, with (α) and (β) shows that for the years 1884–1891 the solar observations have been fairly distributed amongst the observers over the different parts of the orbit. If the constants x and y are neglected, the errors $+0''\cdot20 + 0''\cdot25 \cos\theta$, see equations 14, 24, and 27, are thrown upon the solar tables.

When the method of personal corrections is adopted, it will be necessary from time to time to examine these corrections by a rediscussion of the "errors of ecliptic polar distance" of the tables of the Sun deduced from the observations of the different observers.

This method of applying personal corrections, found independently of the errors $\delta\theta$, to the right ascensions of the Sun appears sound in theory, and advantageous in practice at those observatories where only two or three observers are engaged upon the work, and when the separation of x for each observer from the mean errors of the tables in longitude can, therefore, be practically effected.

There is one point connected with these differences of habit in observing the transits of stars and the limbs of the Sun which appears to deserve notice.

The corrections x which we have found refer strictly to the observations of the Sun in right ascension, and they will not be identical with those required by the adopted R.A. of the clock stars if the observer has different habits of observing the transits of the limbs of the Sun and stars. When, therefore, the epoch corrections are computed as usual from the equation

$$\delta\alpha^s = \frac{x''}{15},$$

such personalities of the observer, or observers, are necessarily thrown upon the fundamental right ascensions of the clock stars, and this is true whether all the observations of the Sun are made by one person or by several.

In the case of the three Oxford observers, the epoch-corrections thus computed for each observer are

$$\text{W.} = +0^s{\cdot}074\,;\ \text{R.} = -0^s{\cdot}077\,;\ \text{F.B.} = +0^s{\cdot}063.$$

The weighted mean is $+0^s{\cdot}013$, and this agrees very closely indeed with that which I used—viz. $+0^s{\cdot}014$—in the formation of the "Radcliffe Catalogue of 6,424 Stars for 1890." But this correction involves the effects of the mean personalities of the three observers as well as any constant error of the adopted right ascensions of the clock stars used for the determination of the errors of the clock.

We have no direct means, so far as I know, of deciding whether or no any particular observer has any personalities of the kind under consideration. We cannot, therefore, directly eliminate such effects of personality from our work. But we have a choice of methods. We can accept the mean habit of several observers as probably less affected by such personalities and more stable than the habit of any one observer, or we can select some standard observer and attempt to adopt his habits as free from error or as giving rise only to a constant error in our work. I am strongly of opinion that on fundamental points it is much safer to trust to the mean habit of several good observers than to that of one; in fact, the selected standard observer, practically, will change with age, and it will be exceedingly difficult to connect his past and present habits with those of other observers.

The solar observations discussed in the present paper afford, in my opinion, a rather striking example of the great reliability of means.

The observations at Oxford 1884–1891 were based on the Greenwich clock-star lists. The errors of the tables in longitude given by the separate observers, W., R., and F.B., uncorrected for personalities, differ systematically and very sensibly from each other; but if we take the mean error in longitude found directly from the observations of all the observers combined, both at Oxford and Greenwich, for the errors of the tables in mean longitude we have, 1884–1891:

Oxford	$+0''{\cdot}765$,
Greenwich	$+0''{\cdot}758$.

The two results are therefore practically identical. This is, of course, to some extent a matter of chance; but the errors of the tables given by the Oxford observations would have differed very sensibly from those given by the Greenwich observers had I rejected, underweighted, or corrected Mr. Robinson's observations

simply because his results differed from the observations of Messrs. Wickham and Bellamy. There is no sensible difference in the habits of the three observers W., R., and F.B. in taking transits of the first and second limbs of the Sun. To show that such is the case the corrections to the tabular semidiameter found from the observations of the different observers are given in Table (δ). The mean correction for the time of passage of the semidiameter is $+0^{s}·063$, and this result does not differ by quite a hundredth of a second from the three separate results for the different observers. There are very slight indications of personal differences in the semidiameters in N.P.D.; but these differences are much too small and uncertain to render any attempts at correction necessary or desirable. But on the effects of personalities in the N.P.D. observations see the values of y given in Table (α).

Radcliffe Observations, 1884–1891.

Errors of Tabular Semidiameter of the Sun for each Observer.

TABLE (δ).

	R.A.						N.P.D.					
	W.		R.		F.B.		W.		R.		F.B.	
Year.	s	No. of Obs.	s	No. of Obs.	s	No. of Obs.	″	No. of Obs.	″	No. of Obs.	″	No. of Obs.
1884	+0·064	25	+0·069	34	+0·063	35	−0·36	26	+0·29	33	+0·49	39
1885	+0·074	30	+0·049	24	+0·067	25	+0·20	31	+0·28	26	+0·23	30
1886	+0·074	22	+0·065	26	+0·072	21	+0·34	22	+0·13	20	+0·29	23
1887	+0·057	18	+0·023	20	+0·080	14	+0·24	17	+0·29	18	+0·15	17
1888	+0·059	10	+0·045	8	+0·065	11	+0·54	11	+0·05	9	+0·53	12
1889	+0·050	10	+0·037	25	+0·076	23	−0·10	10	+0·11	25	+0·19	20
1890	+0·035	19	+0·067	24	+0·041	11	−0·06	16	+0·04	22	+0·33	9
1891	+0·072	22	+0·070	28	+0·086	23	+0·34	22	+0·34	27	+0·73	24
Weighted Means	+0·063	156	+0·056	189	+0·070	163	+0·13	155	+0·21	180	+0·38	174

The existence of well-marked differences in the observations of the transits of the Sun has induced me to try whether the same difference of habit exists in the case of the Moon. On account of the comparatively much larger errors of the Lunar Tables, and the rapidity with which they change, we cannot expect that the application of the corrections

$$W. = +0^{s}·074 \qquad R. = -0^{s}·077 \qquad F.B. = +0^{s}·063$$

to the observed right ascensions will give the same mean errors in longitude for the different observers unless a large number of observations of the Moon, extending over many years, are included in the discussion. But with a large number of observa-

tions the effects of the real errors of the tables should not seriously affect the differences of the errors of the tables thus found for each observer. This test has been applied to the lunar observations made in the years 1884–1891, and the results thus obtained are given in the following Table (ϵ).

Radcliffe Observations of the Moon, 1884–1891.

Mean Errors of the Tables in Longitude for each Observer.

(*Tabular — Observed.*)

TABLE (ϵ).

Year.	Observer.	Errors of the Tables in Longitude.	Errors of the Tables in Longitude Corrected for Personal Values of x.	No. of Observations.
		"	"	
1884	W.	−0·15	−1·19	12
	R.	−3·21	−2·13	27
	F. B.	−0·30	−1·18	22
1885	W.	−1·19	−2·23	13
	R.	−3·85	−2·76	22
	F. B.	+0·25	−0·63	22
1886	W.	−1·65	−2·69	17
	R.	−3·49	−2·41	21
	F. B.	−0·30	−1·18	11
1887	W.	−3·17	−4·21	9
	R.	−4·19	−3·11	27
	F. B.	−0·99	−1·87	18
1888	W.	−1·49	−2·53	11
	R.	−3·41	−2·33	19
	F. B.	−1·45	−2·33	14
1889	W.	−2·91	−3·95	18
	R.	−4·56	−3·48	27
	F. B.	−2·11	−3·00	17
1890	W.	−2·21	−3·25	12
	R.	−4·89	−3·81	22
	F. B.	−2·45	−3·33	12
1891	W.	−1·45	−2·49	15
	R.	−4·18	−3·10	32
	F. B.	−1·95	−2·83	18
Weighted Means 1884–1891	W.	−1·79	−2·83	107
	R.	−3·99	−2·91	197
	F. B.	−1·07	−1·95	134

These results are the errors of Hansen's Tables, corrected for δt, as given in the volumes of the *Radcliffe Observations*, and with tabular semidiameters corrected by $-0^{s}\cdot06$.

It will be seen that the application of the personal equations deduced from the observations of the Sun has greatly reduced the differences of the errors of the Lunar Tables deduced from the observations of the Moon by the different observers.

As a further check upon the sensible uniformity of the habits of the observers in taking transits of the limbs of the Sun and Moon I have had the errors of the tables of the Moon in R.A. for both limbs collected for the different observers. The results uncorrected for personalities are given in Table (ζ), and the results corrected for the personalities, found from the observations of the Sun, are given in Table (η).

Mean Errors of the Tables of the Moon in R.A. for each Observer (Tabular – Observed), uncorrected for x.

TABLE (ζ).

Year.	☽ 1L. W. s	No. of Obs.	☽ 1L. R. s	No. of Obs.	☽ 1L. F.B. s	No. of Obs.	☽ 2L. W. s	No. of Obs.	☽ 2L. R. s	No. of Obs.	☽ 2L. F.B. s	No. of Obs.
1884	−0·068	9	−0·250	18	−0·066	14	+0·203	3	−0·142	9	+0·132	6
1885	−0·103	7	−0·293	15	−0·063	12	−0·070	5	−0·181	7	+0·126	10
1886	−0·142	13	−0·248	17	−0·070	5	+0·078	4	−0·040	3	+0·050	5
1887	−0·257	7	−0·326	18	−0·196	11	+0·055	2	−0·189	9	+0·180	6
1888	−0·175	6	−0·263	11	−0·066	9	+0·028	5	−0·175	8	−0·058	5
1889	−0·212	9	−0·383	15	−0·117	10	−0·160	7	−0·205	11	−0·114	7
1890	−0·267	7	−0·346	11	−0·201	9	+0·103	3	−0·324	11	+0·027	3
1891	−0·217	7	−0·337	20	−0·178	9	0·000	8	−0·197	11	−0·046	8
Weighted Means	−0·174	65	−0·305	125	−0·118	79	0·000	37	−0·199	69	+0·040	50

Corrected for Personal Values of x.

TABLE (η).

Year.	☽ 1L. W. s	No. of Obs.	☽ 1L. R. s	No. of Obs.	☽ 1L. F.B. s	No. of Obs.	☽ 2L. W. s	No. of Obs.	☽ 2L. R. s	No. of Obs.	☽ 2L. F.B. s	No. of Obs.
1884	−0·142	9	−0·173	18	−0·129	14	+0·129	3	−0·065	9	+0·069	6
1885	−0·177	7	−0·216	15	−0·126	12	−0·144	5	−0·104	7	+0·063	10
1886	−0·216	13	−0·171	17	−0·133	5	+0·004	4	+0·037	3	−0·013	5
1887	−0·331	7	−0·249	18	−0·259	11	−0·019	2	−0·112	9	+0·117	6
1888	−0·249	6	−0·186	11	−0·129	9	−0·046	5	−0·098	8	−0·121	5
1889	−0·286	9	−0·306	15	−0·180	10	−0·234	7	−0·128	11	−0·177	7
1890	−0·341	7	−0·269	11	−0·264	9	+0·029	3	−0·247	11	−0·036	3
1891	−0·291	7	−0·260	20	−0·241	9	−0·074	8	−0·120	11	−0·109	8
Weighted Means	−0·248	65	−0·228	125	−0·181	79	−0·074	37	−0·122	69	−0·023	50

The errors of the Lunar Tables are so large, and change so rapidly, that a complete elimination of their effects from these results can hardly be expected. But the much closer agreement of the corrected results for the different observers is quite evident.

If we take the means of the results for the two limbs for each observer to eliminate the effects of the tabular semi-diameter, the correction to which is not very accurately determined, we have

$$\text{W.} = -0^{s}\cdot161\ ;\ \text{R.} = -0^{s}\cdot175\ ;\ \text{F.B.} = -0^{s}\cdot102.$$

These results agree very closely; but they appear to show, like the solar observations—see (27)—that the value of x found from Mr. Bellamy's observations in Ecliptic Polar Distance is rather smaller than the true value.

In the present paper attention has been directed to the existence of very sensible systematic differences in the habits of three skilled observers in taking transits of stars and of limbs of the Sun and Moon. The theory of the effects of such personalities has been discussed, and very approximate determinations of the values of these personalities have been found from a discussion of the solar observations, independently of any errors of the Solar Tables in longitude; and it has been shown that no sensible differences in the habits of the observers exist in taking transits of the two limbs of the Sun. These personal corrections have been shown to be applicable to the case of the Moon as well as of the Sun. The errors of the Solar and Lunar Tables are, by the adoption of these corrections, sensibly freed from the effects of the personalities in the observers; and they differ sensibly from those which are usually accepted as the errors of the tables when the corrections, x, are neglected.

It would appear that a very sensible increase in the accuracy of our determinations of the errors of Solar and Lunar Tables should result from this method of finding the personal corrections, x, and their use as corrections to the observed R.A.

Preliminary Note on a Personal Equation depending on Magnitude affecting the Right Ascensions of the Stars in the Cambridge Zone Catalogue of the Astronomische Gesellschaft, and its determination from Astrographic Catalogue Plates. By Arthur R. Hinks, B.A.

(*Communicated by Sir R. S. Ball.*)

The Cambridge Zone Catalogue was completed by Mr. Graham at the end of 1896. In accordance with the programme of the Gesellschaft the Cambridge zone overlaps the northern Berlin zone (Dr. Becker's) by 20′. There are thus a number of stars common to the two catalogues, and a comparison between their places in the two affords a means of determining any systematic error which is not common to both.

With the assent of the director I undertook to make the comparison as part of my regular work at the Cambridge Observatory, and the results of this and the subsequent work suggested by it form the substance of this note.

The zone $+24° 50'$ to $+25° 10'$ is common to the Cambridge and Berlin zones, and 671 stars, mainly within these limits, are found in both catalogues. A table was formed giving the difference (Cambridge − Berlin) of the catalogue right ascensions and declinations of each star. It was immediately evident that the differences in R.A. were small, about $0^s\!\cdot07$ on the average, but were in general positive, and that there was thus a systematic difference between the catalogues. The differences in declination were small, about $0''\!\cdot5$ on an average, and with them we are not concerned in this note. They appeared to be mainly accidental.

The most probable cause of such a discrepancy appeared to be the existence of a personal equation depending on magnitude affecting one or more probably both series of observations. Several of the observatories co-operating in the scheme of the Astronomische Gesellschaft had already investigated this question in reference to their own observations, and determined a magnitude equation of sensible amount. The Cambridge Catalogue is particularly well adapted to furnish evidence of value on this point, since by far the greater part of the R.A. observations were made by one and the same observer, Mr. Graham. The results should, therefore, be homogeneous so far as personal equation is concerned.

Before, however, discussing the differences with reference to personal equation, it is necessary to satisfy ourselves as far as possible that no other causes may have contributed to the observed discrepancy. To do this we must glance at the circumstances under which the observations were made.

The work on the zone catalogue was begun at Cambridge in the summer of 1872. Until the autumn of 1883 the stars were